Ex 1

Esempi

- $M = 0.999 \cdot 10^{12} \in \mathcal{F} \Rightarrow M+M = 0.1998\ 10^{13} \notin \mathcal{F}$
- $m_1 = 0.100 \cdot 10^{-12}$ $m_2 = 0.101 \cdot 10^{-12} \Rightarrow m_2 - m_1 = 0.001 \cdot 10^{-12} = 0.1\,10^{-14} \notin \mathcal{F}$
- $\text{Spacing}_{[\beta^p, \beta^{p+1}]} = \beta^{p+1-t} = 10^{p-2} = 10$ $\quad p = 3 \Rightarrow [10^3, 10^4]$ interi non consecutivi
  $t=3$
  $\beta=10$
  $\Rightarrow x = 1001 = 0.1001 \cdot 10^4 \notin \mathcal{F}$
- $1, 7 \in \mathcal{F}$
  $\frac{1}{7} = 0.1428\ldots \xrightarrow[\text{per eccesso}]{fl} 0.143$ $\qquad 0.143 * 7 = 0.1001 \cdot 10^1 \xrightarrow[\text{per difetto}]{fl} 0.1 \cdot 10^1 \in \mathcal{F}$
  $1, 8 \in \mathcal{F}$
  $\frac{1}{8} = 0.125 \in \mathcal{F}$ $\qquad 0.125 * 8 = 0.1 \cdot 10^1 \in \mathcal{F}$

Ex2

- $A_2 = M_1 A = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{\alpha_2}{\alpha_1} & 1 & 0 \\ -\frac{\alpha_3}{\alpha_1} & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_2 & \beta_2 & 0 \\ \alpha_3 & 0 & \beta_3 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & -\frac{\alpha_2^2}{\alpha_1} + \beta_2 & -\frac{\alpha_2\alpha_3}{\alpha_1} \\ 0 & -\frac{\alpha_2\alpha_3}{\alpha_1} & -\frac{\alpha_3^2}{\alpha_1} + \beta_3 \end{bmatrix}$ –

$\alpha_1 \neq 0$

$$a = -\frac{\alpha_2^2}{\alpha_1} + \beta_2$$

$$b = -\frac{\alpha_2\alpha_3}{\alpha_1}$$

$$\frac{b}{a} = +\frac{\alpha_2\alpha_3}{\cancel{\alpha_1}} \frac{\cancel{\alpha_1}}{-\alpha_2^2 + \alpha_1\beta_2} = \frac{\alpha_2\alpha_3}{-\alpha_2^2 + \alpha_1\beta_2}$$

$$A_3 = M_2 A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{b}{a} & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & a & b \\ 0 & b & -\frac{\alpha_3^2}{\alpha_1} + \beta_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & a & b \\ 0 & 0 & c \end{bmatrix}}_{= U}$$

$a \neq 0$

$$d = -\frac{b}{a} b - \frac{\alpha_3^2}{\alpha_1} + \beta_3$$

$$= \beta_3 - \frac{\alpha_3^2}{\alpha_1} - \frac{\alpha_2^2\alpha_3^2}{\alpha_1^2} \frac{1}{\beta_2 - \frac{\alpha_2^2}{\alpha_1}}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\alpha_2}{\alpha_1} & 1 & 0 \\ \frac{\alpha_3}{\alpha_1} & \frac{b}{a} & 1 \end{bmatrix}$$

- $L \cdot U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{\alpha_3}{\beta_3} & \frac{\alpha_2}{\beta_2} & 1 \end{bmatrix} \begin{bmatrix} \beta_3 & 0 & \alpha_3 \\ 0 & \beta_2 & \alpha_2 \\ 0 & 0 & \alpha_1 - \frac{\alpha_2^2}{\beta_2} - \frac{\alpha_3^2}{\beta_3} \end{bmatrix} = \begin{bmatrix} \beta_3 & 0 & \alpha_3 \\ 0 & \beta_2 & \alpha_2 \\ \alpha_3 & \alpha_2 & \alpha_1 \end{bmatrix}$

$$\Rightarrow \quad P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

• $A = A^T \quad \forall \alpha_1 \; \alpha_2 \; \alpha_3 \quad \beta_2 \; \beta_3$

Criterio di Sylvester

$\alpha_1 > 0$ $\qquad\qquad$ $\beta_2, \beta_3 > 0$

$$\begin{vmatrix} \alpha_1 & \alpha_2 \\ \alpha_2 & \beta_2 \end{vmatrix} = \alpha_1\beta_2 - \alpha_2^2 > 0$$

$$\det A = \alpha_3 \begin{vmatrix} \alpha_2 & \alpha_3 \\ \beta_2 & 0 \end{vmatrix} + \beta_3 \begin{vmatrix} \alpha_1 & \alpha_2 \\ \alpha_2 & \beta_2 \end{vmatrix} = -\alpha_3^2\beta_2 + \beta_3(\alpha_1\beta_2 - \alpha_2^2) > 0$$

• $R = D^{-1/2} U$ con U della fattorizzazione LU precedentemente calcolata

$$D^{-1/2} = \operatorname{diag}\left(\frac{1}{\sqrt{u_{ii}}}\right) = \operatorname{diag}\left(\frac{1}{\sqrt{\alpha_1}}, \frac{1}{\sqrt{a}}, \frac{1}{\sqrt{c}}\right)$$

$$\Rightarrow \quad R = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{\alpha_1}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{a}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{c}} \end{bmatrix}}_{\text{moltiplica righe}} \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & a & b \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} \sqrt{\alpha_1} & \frac{\alpha_2}{\sqrt{\alpha_1}} & \frac{\alpha_3}{\sqrt{\alpha_1}} \\ 0 & \frac{a}{\sqrt{a}} & \frac{b}{\sqrt{a}} \\ 0 & 0 & \frac{c}{\sqrt{c}} \end{bmatrix} = \begin{bmatrix} \sqrt{\alpha_1} & \frac{\alpha_2}{\sqrt{\alpha_1}} & \frac{\alpha_3}{\sqrt{\alpha_1}} \\ 0 & \sqrt{a} & \frac{b}{\sqrt{a}} \\ 0 & 0 & \sqrt{c} \end{bmatrix}$$

$$\frac{a}{\sqrt{a}} = \sqrt{a} = \sqrt{-\frac{\alpha_2^2}{\alpha_1} + \beta_2} \qquad\qquad \frac{b}{\sqrt{a}} = -\frac{\alpha_2\alpha_3}{\alpha_1} \; \frac{1}{\sqrt{\beta_2 - \frac{\alpha_2^2}{\alpha_1}}} =$$

$$\frac{c}{\sqrt{c}} = \sqrt{c}$$

Ex3

$$f^*(x) = c^* \quad \text{t.c.} \quad \sum_{k=0}^{1} \left(f(x_k) - c^*\right)^2 \le \sum_{k=0}^{1} \left(f(x_k) - c\right)^2$$

$m = 1 \quad k = 0, m$

$\{\phi_j\}_{j=0,n} = \{1\} \implies$ sistema eq normali

$n = 0$

$(x_0, y_0) = (a, f(a))$

$(x_1, y_1) = (b, f(b))$

$$C \underline{a} = \underline{b} \qquad C_{ij} = \sum_{k=0}^{m} \phi_i(x_k)\phi_j(x_k) \qquad \underline{a} = [c^*]$$

$$b_i = \sum_{k=0}^{m} f(x_k)\,\phi_i(x_k)$$

$$\implies C_{11}\,\underline{a} = b_1 \qquad C_{11} = 1 + 1 = 2$$

$$b_1 = f(a) + f(b)$$

$$\implies \underline{a} = [c^*] = \left[\frac{f(a)+f(b)}{2}\right]$$

$$\tilde{I}(f) = \int_a^b c^*\,dx = \frac{f(a)+f(b)}{2}\int_a^b dx = \frac{b-a}{2}\left(f(a)+f(b)\right)$$

metodo dei Trapezi

$\implies$ grado di precisione $k = 1$

EX 4

$$K_1 = f(t_n, y(t_n))$$

$$K_2 = f(t_n + hc_2, y(t_n) + hc_2 K_1) = f(t_n + hc_2, y(t_n) + \underline{hc_2} f(t_n, y(t_n))) =$$

$$= f(t_n, y(t_n)) + hc_2 \frac{\partial f}{\partial t}(t_n, y(t_n)) + hc_2 \frac{\partial f}{\partial y}(t_n, y(t_n)) + O(h^2)$$

Taylor in t_n

$$\Rightarrow y_{n+1} \approx y(t_n) + hb_1 f(t_n, y(t_n)) + hb_2 \left(f(t_n, y(t_n)) + hc_2 \frac{\partial f}{\partial t}(t_n, y(t_n)) + hc_2 \frac{\partial f}{\partial y}(t_n, y(t_n)) \right) + O(h^3)$$

$$= y(t_n) + h(b_1 + b_2) f(t_n, y(t_n)) + h^2 b_2 c_2 \left(\frac{\partial f}{\partial t}(t_n, y(t_n)) + \frac{\partial f}{\partial y}(t_n, y(t_n)) \right) + O(h^3)$$

$$y(t_{n+1}) = y(t_n) + h y'(t_n) + \frac{h^2}{2} y''(t_n) + O(h^3) = y(t_n) + h f(t_n, y(t_n)) + \frac{h^2}{2} \left(\frac{\partial f}{\partial t}(t_n, y(t_n)) + \frac{\partial f}{\partial y}(t_n, y(t_n)) \right) + O(h^3)$$

$$y' = f(t, y)$$

$\Rightarrow$ per la consistenza deve essere

$$b_1 + b_2 = 1$$

$$b_2 c_2 = \frac{1}{2}$$

Ci sono ∞ metodi consistenti

NB Essendo un metodo ad un passo, se è consistente è convergente.